B-Math-III Supplementary Exam ; Differential Equations.

Time : 2.30 hrs; Max Mark: 60 ; June 2022

1. Suppose that the steady state temperature distribution $u(x, y)$ on a semi-infinite strip $S := \{(x, y) : 0 \leq x \leq l, 0 \leq y < \infty\}$ has the boundary conditions $u(0, y) = u(l, y) = 0, 0 < y < \infty$ and $u(x, 0) = f(x), 0 < x < l$. Find $u(x, y)$ in S. (15)

2. Find the general solution of Legendre's equation $(1-x^2)y''-2xy'+p(p+1)y = 0$, in a neighbourhood of $x = 0$, where $p \in \mathbb{R}$. (10)

3. Find two linearly independent solutions of Bessel's equation
$x^2y'' + xy' + (x^2 - p^2)y = 0$ in the region $x > 0$ when $p > 0$ is not an integer. (15)

4. Suppose that the characteristic ODEs of an unknown first order, quasi-linear PDE with solution $u(x, y)$ are given as

$$\frac{dx}{dt} = t + \frac{s}{2}; \frac{dy}{dt} = 1; \frac{du}{dt} = 1,$$

with the initial curve $\Gamma_0 : \{(x, y); x = s, y = s, 0 \leq s \leq 1\}$, and initial value $u = \frac{s}{2}, 0 \leq s \leq 1$. Find the PDE and solve it. (15)

5. Let $y(x), x > 0$ be a solution of the differential equation $x^2y''+xy'+(x^2-1)y = 0$. let $u(x), x > 0$ be a solution of $u'' + (1 - \frac{3}{4x^2})u = 0$. Show that $y(x) = C\frac{u(x)}{\sqrt{x}}$ for some constant $C > 0$. Hence deduce that $y(x)$ has atmost one zero in the interval $(0, \frac{\pi}{4})$. (8+7)